

DPP-1

(SOLUTIONS)

Q.1 If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then $P(x) \cdot Q(x) = 0$ has

- (A) exactly one real root (B) atleast two real roots
(C) exactly three real roots (D) all four are real roots.

Sol. $D_1: b^2 - 4ac$ and $D_2: d^2 + 4ac$. Hence atleast one of either D_1 or D_2 is zero]

Q.2 If α, β are the roots of the equation $x^2 + px - r = 0$ and $\frac{\alpha}{3}, 3\beta$ are the roots of the equation $x^2 + qx - r = 0$, then r equals

- (A) $\frac{3}{8}(p - 3q)(3p + q)$ (B) $\frac{3}{8}(p + 3q)(3p - q)$
(C) $\frac{3}{64}(3p - q)(p - 3q)$ (D) $\frac{3}{64}(3q - p)(p - q)$

Sol. Given, $x^2 + px - r = 0$ $\begin{cases} \alpha \\ \beta \end{cases}$

$$\therefore \alpha + \beta = -p \quad \dots(1)$$

$$\text{and } \alpha\beta = -r \quad \dots(2)$$

Also, $x^2 + qx - r = 0$ $\begin{cases} \frac{\alpha}{3} \\ 3\beta \end{cases}$

$$\therefore \frac{\alpha}{3} + 3\beta = -q \Rightarrow \alpha + 9\beta = -3q \quad \dots(3)$$

$$\text{and } \left(\frac{\alpha}{3}\right)(3\beta) = -r \Rightarrow \alpha\beta = -r$$

So, on solving (1) and (3), we get

$$\alpha = \frac{3(q - 3p)}{8}, \beta = \frac{(p - 3q)}{8}$$

$$\text{As, } \alpha\beta = -r$$

$$\Rightarrow r = \frac{3(3p - q)(p - 3q)}{64}$$

Q.3 Let λ_1 and λ_2 be two values of λ for which the expression $x^2 + (2 - \lambda)x + \lambda - \frac{3}{4}$ becomes a perfect square. The value of $(\lambda_1^2 + \lambda_2^2)$ equals

- (A) 8 (B) 25 (C) 50 (D) 100

Sol. Given, $f(x) = x^2 + (2 - \lambda)x + \left(\lambda - \frac{3}{4}\right)$

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Now, for a quadratic expression to become a perfect square, its $D = 0$

$$\Rightarrow (2 - \lambda)^2 = 4 \left(\lambda - \frac{3}{4} \right) \Rightarrow 4 + \lambda^2 - 4\lambda = 4\lambda - 3 \Rightarrow \lambda^2 - 8\lambda + 7 = 0 \Rightarrow (\lambda - 7)(\lambda - 1) = 0$$

$$\therefore \lambda = 1, 7$$

$$\text{Hence, } (\lambda_1^2 + \lambda_2^2) = (1)^2 + (7)^2$$

$$= 1 + 49 = 50. \text{ Ans.]}$$

Q.4 The value of the expression $x^4 - 8x^3 + 18x^2 - 8x + 2$ when $x = \cot \frac{\pi}{12}$, is

(A) 2

(B) 1

(C) 0

(D) 3

Sol. As $\cot \frac{\pi}{12} = \cot 15^\circ = 2 + \sqrt{3} = x$ (say) $\Rightarrow (x - 2)^2 = 3 \Rightarrow x^2 - 4x + 1 = 0 \Rightarrow x^2 = 4x - 1$

$$\text{Now, expression} = x^4 - 8x^3 + 18x^2 - 8x + 2 = x^2(4x - 1) - 8x^3 + 18x^2 - 8x + 2$$

$$= -4x^3 + 17x^2 - 8x + 2 = -4x(4x - 1) + 17x^2 - 8x + 2$$

$$= x^2 - 4x + 2 = (4x - 1) - 4x + 2 = -1 + 2 = 1. \text{ Ans.]}$$

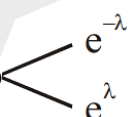
Q.5 If e^λ and $e^{-\lambda}$ are the roots of equation $3x^2 - (a + b)x + 2a = 0$, $a, b, \lambda \in \mathbb{R}, \lambda \neq 0$ then least integral value of b is

(A) 4

(B) 5

(C) 9

(D) 10

Sol. $f(x) = 3x^2 - (a + b)x + 2a = 0$ 

$$\text{Product of the roots} = 1 \Rightarrow \frac{2a}{3} = 1 \Rightarrow a = \frac{3}{2}$$

$$\text{Now, sum of the roots} = e^{-\lambda} + e^\lambda = \frac{a+b}{3} > 2 \Rightarrow b > 6 - a$$

$$\Rightarrow b > 6 - \frac{3}{2} \Rightarrow b > \frac{9}{2}$$

$\therefore$ Least value of b is 5.

Q.6 Let x_1 and x_2 be the roots of the quadratic equation $x^2 + px + q = 0$.

If $x_1 = \frac{x_2 + 4}{2x_2 - 1}$, then the value of $(2q + p)$ is equal to

(A) 1

(B) 2

(C) 3

(D) 4

Sol. $x_1 + x_2 = -p$; $x_1 x_2 = q$

$$\text{Given } 2x_1 x_2 - x_1 = x_2 + 4$$

$$2x_1 x_2 - (x_1 + x_2) = 4 \quad 2q + p = 4 \text{ Ans.]}$$

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Q.7 If α, β are the roots of the quadratic equation $x^2 + 2(1 - \cos 3\theta)x - 2\sin^2 3\theta = 0$ ($\theta \in \mathbb{R}$), then the maximum value of $\alpha^2 + \beta^2$ is equal to

- (A) 0 (B) 4 (C) 8 (D) 16

Sol. We have $x^2 + 2(1 - \cos 3\theta)x - 2\sin^2 3\theta = 0$
 $\alpha \quad \beta$

$$\alpha + \beta = -2(1 - \cos 3\theta); \alpha\beta = -2\sin^2 3\theta$$

$$\text{Now, } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4(1 - \cos 3\theta)^2 + 4\sin^2 3\theta = 4(1 - 2\cos 3\theta + \cos^2 3\theta + \sin^2 3\theta)$$

$$\Rightarrow \alpha^2 + \beta^2 = 4(2 - 2\cos 3\theta)$$

Clearly, maximum value of $\alpha^2 + \beta^2$ is 16 when $\cos 3\theta = -1$.]

Q.8 If $(a + b + c) > 0$ and $a < 0 < b < c$, then the equation $a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$ has

- (A) real and distinct roots (B) roots are imaginary
 (C) product of roots is negative (D) product of roots is positive

Sol. Let $P(x) = a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$

Now, $P(0) = 3abc$ which is negative

Hence, product of the roots is negative

$$\text{Now, } P(a) = a(a - b)(a - c) < 0$$

$$P(b) = b(b - c)(b - a) < 0$$

$$P(c) = c(c - a)(c - b) > 0$$

Hence, roots are real and distinct $\Rightarrow$ (A) and (C).

Aliter: $a + b + c)x^2 - 2(ab + bc + ca)x + 3abc = 0$ $D = 4[(ab + bc + ca)^2 - 3(abc)(a + b + c)]$
 Since, $a + b + c > 0$ and $abc < 0$

$$\therefore D > 0$$

$$\text{Product of roots} = \frac{3abc}{a+b+c} < 0.]$$

Q.9 If one of the root of the equation $4x^2 - 15x + 4p = 0$ is the square of the other then the value of p is

- (A) 125/64 (B) -27/8 (C) -125/8 (D) 27/8

Sol. If α is one root then

$$\alpha + \alpha^2 = 15/4 \text{ and } \alpha^3 = p$$

$$4\alpha^2 + 4\alpha - 15 = 0$$

$$4\alpha^2 + 10\alpha - 6\alpha - 15 = 0$$

$$2\alpha(2\alpha + 5) - 3(2\alpha + 5) = 0 \Rightarrow \alpha = -5/2 \text{ or } \alpha = 3/2$$

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Q.10 Consider the quadratic equation $(k^2 - 1)x^2 + (2k^3 + 9k^2 + 3k - 14)x + (2k^3 + 5k^2 - 11k - 14) = 0$ then find the sum of all the value(s) of k (where $k \in \mathbb{R}$) for which the given equation has

Column-I

- (A) Exactly one root zero and other root is finite
- (B) Both roots zero
- (C) Exactly one root infinity
- (D) Both roots infinity

Column-II

- (P) -1
- (Q) $\frac{-5}{2}$
- (R) 2
- (S) $\frac{-7}{2}$
- (T) 1

Sol. $(k - 1)(k + 1)x^2 + (2k + 7)(k - 1)(k + 2)x + (2k + 7)(k - 2)(k + 1) = 0$

- (A) Exactly one root is '0' and other root is finite.

$$\text{P.O.R.} = 0 \text{ but S.O.R.} \neq 0$$

$$\text{Therefore } k = 2.$$

- (B) Both roots are zero means S.O.R. = P.O.R. = 0

$$\text{Hence, } k = \frac{-7}{2} \text{ only}$$

- (C) Exactly one root infinity

$$\text{i.e. coefficient of } x^2 = 0 \text{ but coefficient of } x \neq 0$$

$$\text{Therefore } k = -1 \text{ only.}$$

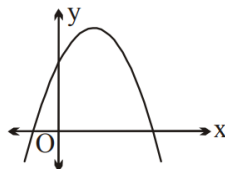
- (D) Both roots are infinity

$$\text{i.e. coefficient of } x^2 = \text{coefficient of } x = 0 \text{ but constant term} \neq 0$$

$$\text{Therefore } k = 1 \text{ only.]}$$

SOLUTIONS

Q.1 The graph of a quadratic polynomial $y = px^2 - qx + r$ is as shown in the adjacent figure, then



- (A) $r^2 - 4q < 0$ (B) $r^2 - 4p < 0$ (C) $p + q > 0$ (D) $r - p - q > 0$

Sol. $r > 0$; $p < 0$ & $q < 0$. Now verify it.]

Q.2 The smallest integral value of p for which the inequality $(p - 3)x^2 - 2px + 3(p - 2) > 0$ is satisfied for all real values of x , is

- (A) 8 (B) 7 (C) 6 (D) 5

Sol. We have, $(p - 3)x^2 - 2px + 3(p - 2) > 0$ is true for all $x \in \mathbb{R}$, so

$$p - 3 > 0$$

$$\text{and } \text{Disc.} < 0$$

must be satisfied simultaneously.

$$\therefore p > 3$$

$$\text{and } D < 0 \Rightarrow 4p^2 - 4(p - 3)(3p - 6) < 0$$

$$\Rightarrow 2p^2 - 15p + 18 > 0$$

$$\Rightarrow 2p^2 - 12p - 3p + 18 > 0$$

$$\Rightarrow 2p(p - 6) - 3(p - 6) > 0$$

$$\Rightarrow (2p - 3)(p - 6) > 0$$

$$\Rightarrow p > 6 \text{ or } p < \frac{3}{2}$$

$$\therefore (1) \cap (2) \Rightarrow p \in (6, \infty)$$

Hence, the smallest integral value of p equals 7. Ans.]

Q.3 The complete solution set of the inequality $\frac{3^x(2x-5)(x^2+x+2)}{(\cos x - 2)(x^2+x)} \leq 0$ is

- (A) $(-\infty, -1)$ (B) $(\frac{5}{2}, \infty)$ (C) $(-1, \frac{5}{2}]$ (D) $(-1, 0) \cup [\frac{5}{2}, \infty)$

Sol. $\frac{(2x-5)}{x(x+1)} \geq 0$

Q.4 If the graph of $f(x) = x^2 + (3 - k)x + k$, (where $k \in \mathbb{R}$) lies above and below x -axis, then k cannot be

- (A) -1 (B) 0 (C) 1 (D) 10

Sol. $D > 0$ $k \in (-\infty, 1) \cup (9, \infty)$.

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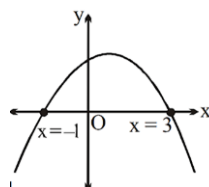
Q.5 The largest integral value of k for which the quadratic trinomial $P(x) = (k - 2)x^2 + 8x + k + 4$ is non-positive for all real values of x is

- (A) 2 (B) 4 (C) -6 (D) -2

Sol. $k - 2 < 0$ and $D \leq 0$

$$k \in (-\infty, -6]]$$

Q.6 Consider the graph of quadratic polynomial $y = ax^2 + bx + c$ as shown below. Which of the following is(are) correct?



- (A) $\frac{a-b+c}{abc} = 0$ (B) $abc(9a + 3b + c) < 0$
 (C) $\frac{a+3b+9c}{abc} < 0$ (D) $ab(a - 3b + 9c) > 0$

Sol. Clearly, from the given figure,

$$a < 0, c > 0. \text{ Also, } \frac{-b}{2a} > 0 \Rightarrow b > 0$$

$$\text{So, } abc < 0$$

$$\text{Also, } f(-1) = a - b + c = 0 \text{ and } f(3) = 9a + 3b + c = 0$$

$$\text{Clearly, } f\left(\frac{1}{3}\right) > 0 \Rightarrow \frac{a}{9} + \frac{b}{3} + c > 0 \Rightarrow a + 3b + 9c > 0.$$

$$\text{Also } f\left(-\frac{1}{3}\right) > 0 \Rightarrow \frac{a}{9} - \frac{b}{3} + c > 0 \Rightarrow a - 3b + 9c > 0$$

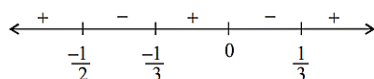
Now, verify alternatives. Ans.]

Q.7 If S is the set of all real ' x ' such that $\frac{x^2(5-x)(1-2x)}{(5x+1)(x+2)}$ is negative and $\frac{3x+1}{6x^3+x^2-x}$ is positive, then S contains

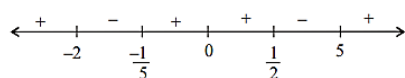
- (A) (1,4) (B) (5,11) (C) $\left(-\frac{3}{2}, \frac{-1}{2}\right)$ (D) $(-10, -4)$

Sol. We have $\frac{3x+1}{x(6x^2+x-1)} > 0$

$$\Rightarrow \frac{3x+1}{x(3x-1)(2x+1)} > 0 \quad \dots\dots\dots(1)$$



$$\text{Now, } \frac{x^2(x-5)(2x-1)}{(5x+1)(x+2)} < 0 \quad \dots\dots\dots(2)$$



$$x \in \left(-2, \frac{-1}{5}\right) \cup \left(\frac{1}{2}, 5\right)$$

Clearly, from (1) & (2), option(s) (A) & (C) are correct.]

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Q.8 Let $f(x) = x^2 + px - 2$, $g(x) = px^2 + x + (p + 2) \forall x \in \mathbb{R}$, where p is a real constant. If $f(x) > g(x) \forall x \in \mathbb{R}$, then the range of p is $(-\infty, -\frac{m}{n})$ where m and n are coprime. Find the value of $(m - 5n)$.

Sol. $f(x) = x^2 + px - 2$

$$g(x) = px^2 + x + p + 2$$

$$f(x) > g(x) \forall x \in \mathbb{R}$$

$$x^2(1 - p) + x(p - 1) - (p + 4) > 0 \forall x \in \mathbb{R}$$

$$1 - p > 0 \text{ and } D < 0$$

$$(p - 1)^2 + 4(1 - p)(p + 4) < 0$$

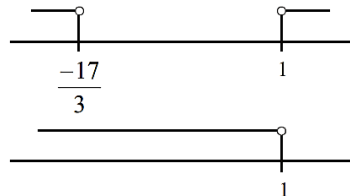
$$(p - 1)[(p - 1) - 4(p + 4)] < 0$$

$$(p - 1)(-3p - 17) < 0$$

$$(p - 1)(3p + 17) > 0$$

$$\therefore p \in (-\infty, -\frac{17}{3}) \equiv (-\infty, -\frac{m}{n})$$

$$\therefore (m - 5n) = 17 - 15 = 2$$



Q.9 Let $x^2 + 2y^2 - 2xy - 2 \geq k(x + 2y) \forall x, y \in \mathbb{R}$ then find the number of integral values of k .

Sol. $x^2 - (2y + k)x + 2y^2 - 2ky - 2 \geq 0 \forall x \in \mathbb{R}$

$$D \leq 0$$

$$(2y + k)^2 - 4(2y^2 - 2ky - 2) \leq 0$$

$$4y^2 + k^2 + 4ky - 8y^2 + 8ky + 8 \leq 0$$

$$-4y^2 + 12ky + k^2 + 8 \leq 0$$

$$4y^2 - 12ky - (k^2 + 8) \geq 0 \forall y \in \mathbb{R}$$

$$D \leq 0$$

$$144k^2 + 4 \cdot 4 \cdot (k^2 + 8) \leq 0$$

$$\therefore k \in \{\emptyset\}$$

Q.10 If the roots of the equation $x^2 + (p + 1)x + 2q - q^2 + 3 = 0$ are real and unequal $\forall p \in \mathbb{R}$ then find the minimum integral value of $(q^2 - 2q)$.

Sol. $x^2 + (p + 1)x + 2q - q^2 + 3 = 0$

$$D > 0$$

$$(p + 1)^2 - 4(2q - q^2 + 3) > 0$$

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$$p^2 + 2p + 1 - 8q + 4q^2 - 12 = 0$$

$$p^2 + 2p + (4q^2 - 8q - 11) > 0 \forall p \in \mathbb{R}$$

$$\Rightarrow 4 - 4(4q^2 - 8q - 11) < 0$$

$$1 - 4q^2 + 8q + 12 < 0$$

$$4q^2 - 8q - 12 > 0$$

$$q^2 - 2q - 3 > 0$$

$$q^2 - 2q > 3$$

$\therefore$ Minimum integral value of $(q^2 - 2q)$ is 4



SOLUTIONS

Q.1 A monic quadratic trinomial $P(x)$ is such that $P(x) = 0$ and $P(P(P(x))) = 0$ have a common root, then

(A) $P(0) \cdot P(1) > 0$

(B) $P(0) \cdot P(1) < 0$

(C) $P(0) \cdot P(1) = 0$

(D) none

Sol. Let $P(x) = x^2 + ax + b$, and let r be a common root.

$\therefore P(r) = 0$ and $P(P(P(r))) = 0$ i.e. $P(P(0)) = 0$, $P(0) = b$

$\therefore P(b) = 0 \Rightarrow b^2 + 4ab + b = 0 \Rightarrow b(a + b + 1) = 0 \Rightarrow P(0) \cdot P(1) = 0.$

Q.2 If two roots of the equation $(x - 1)(2x^2 - 3x + 4) = 0$ coincide with roots of the equation $x^3 + (a + 1)x^2 + (a + b)x + b = 0$ where $a, b \in \mathbb{R}$ then $2(a + b)$ equals

(A) 4

(B) 2

(C) 1

(D) 0

Sol. We have,

$x^3 + (a + 1)x^2 + (a + b)x + b = 0$ (1)

$\Rightarrow (x + 1)(x^2 + ax + b) = 0$ (2)

Also, $(x - 1)(2x^2 - 3x + 4) = 0$ (Given)

As equation (1) and (2) have two roots in common, so two roots of the equation (2) are imaginary. Since $2x^2 - 3x + 4 = 0 \Rightarrow D < 0$

$\therefore$ Equations $x^2 + ax + b = 0$ and $2x^2 - 3x + 4 = 0$ must be identical

Hence, $\frac{1}{2} = \frac{a}{-3} = \frac{b}{4} \Rightarrow a = \frac{-3}{2}$, $b = 2$

$\therefore (a + b) = \frac{-3}{2} + 2 = \frac{1}{2} \Rightarrow 2(a + b) = 1$

Q.3 If $c^2 = 4d$ and the two equations $x^2 - ax + b = 0$ and $x^2 - cx + d = 0$ have one common root, then the value of $2(b + d)$ is equal to

(A) $\frac{a}{c}$

(B) ac

(C) $2ac$

(D) $a + c$

Sol. With the condition, $c^2 = 4d$, the equation $x^2 - cx + d = 0$ has equal roots.

Each root $= \frac{c}{2}$.

As $\frac{c}{2}$ is a root of $x^2 - ax + b = 0$

$\therefore \frac{c^2}{4} - a \cdot \left(\frac{c}{2}\right) + b = 0 \Rightarrow \frac{4d}{4} - \frac{ac}{2} + b = 0 \Rightarrow b + d = \frac{ac}{2} \Rightarrow 2(b + d) = ac$. Ans.]

Q.4 If one root of quadratic equation $x^2 - x + 3m = 0$ is 4 times the root of the equation $x^2 - x + m = 0$, where $m \neq 0$, then m equals

(A) $\frac{12}{196}$

(B) $\frac{12}{169}$

(C) $\frac{12}{256}$

(D) $\frac{12}{189}$

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Sol. $\alpha^2 - \alpha + m = 0$

$$16\alpha^2 - 4\alpha + 3m = 0$$

$\therefore$ On solving, we get

$$\alpha = \frac{13}{12} m$$

On putting above value of α in equation (1), we get

$$m = \frac{12}{169} (\text{As } m \neq 0) \text{ Ans.]}$$

Q.5 If the equations $x^3 + x^2 - 4x = 4$ and $x^2 + px + 2p = 0$ ($p \in \mathbb{R}$) have two roots common, then the value of p is

- (A) -2 (B) -1 (C) 1 (D) 3

Sol. $x^3 + x^2 - 4x - 4 = 0 \Rightarrow x(x^2 - 4) + 1(x^2 - 4) = 0 \Rightarrow (x + 1)(x - 2)(x + 2) = 0$

$$\Rightarrow (x^2 - x - 2)(x + 2) = 0. \text{ So, } p = -1 \text{ Ans.]}$$

Q.6 Statement-1: If the equations $a^2 + bx + c = 0$ ($a, b, c \in \mathbb{R}$ and $a \neq 0$) and $2x^2 + 7x + 10 = 0$ have a common root, then $\frac{2a+c}{b} = 2$.

Statement-2: If both roots of $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ are same, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. Given $a_1, b_1, c_1, a_2, b_2, c_2 \in \mathbb{R}$ and $a_1a_2 \neq 0$.

- (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
 (C) Statement-1 is true, statement-2 is false.
 (D) Statement-1 is false, statement-2 is true.

Sol. As, roots of equation $2x^2 + 7x + 10 = 0$ are non-real, so both roots must be common.

$$\Rightarrow \frac{a}{2} = \frac{b}{7} = \frac{c}{10} = \lambda \text{ (say)}$$

$$\text{So, } a = 2\lambda, b = 7\lambda, c = 10\lambda$$

$$\text{Hence, } \frac{2a+c}{b} = \frac{4\lambda+10\lambda}{7\lambda} = 2.$$

Obviously, S-2 is true and explaining S-1 also. Ans.]

Q.7 If all the equations $x^2 + px + 10 = 0$, $x^2 + qx + 8 = 0$ and $x^2 + (2p + 3q)x + 60 = 0$, where $p, q \in \mathbb{R}$ have a common root, then the value of $(p - q)$ can be

- (A) -1 (B) 0 (C*) 1 (D) 2

Sol. Let α be the common root.

$$\text{So, } \alpha^2 + p\alpha + 10 = 0 \quad \dots\dots\dots(1)$$

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$$\alpha^2 + q\alpha + 8 = 0 \quad \dots\dots\dots(2)$$

$$\alpha^2 + (2p + 3q)\alpha + 60 = 0 \quad \dots\dots\dots(3)$$

Now, $(1) \times 2 + (2) \times 3$

$$\Rightarrow (2\alpha^2 + 2p\alpha + 20) + (3\alpha^2 + 3q\alpha + 24) = 0$$

$$\Rightarrow 5\alpha^2 + (2p + 3q)\alpha + 44 = 0 \quad \dots\dots\dots(5)$$

$\therefore$ Equation (4)-equation (3)

$$\Rightarrow 4\alpha^2 = 16 \Rightarrow \alpha = \pm 2$$

So, for $\alpha = 2, p = -7$ and $q = -6$

$$\Rightarrow (p - q) = -7 - (-6) = -1. \text{ Ans.}$$

Also, for $\alpha = -2, p = 7$ and $q = 6$

$$\Rightarrow (p - q) = 7 - 6 = 1.$$

Q.8 If quadratic equations $2x^2 - 3x + 5 = 0$ and $ax^2 - bx + c = 0, a, b, c \in \mathbb{N}$ have a common root then the value of $a + b + c$ can be equal to

- (A) 10 (B) 15 (C) 20 (D) 25

Sol. $\therefore$ both the equations have both roots are in common

$$\therefore \frac{a}{2} = \frac{-b}{-3} = \frac{c}{5} = \lambda$$

$$\Rightarrow a = 2\lambda, b = 3\lambda, c = 5\lambda$$

$$a + b + c = 10\lambda, \lambda \in \mathbb{N}.]$$

Q.9 The equations $x^3 + 4x^2 + px + q = 0$ and $x^3 + 6x^2 + px + r = 0$ have two common roots, where $p, q, r \in \mathbb{R}$. If their uncommon roots are the roots of equation $x^2 + 2ax + 8c = 0$, then

- (A) $a + c = 8$ (B) $a + c = 2$ (C) $3q = 2r$ (D) $3r = 2q$

Sol. We have, $x^3 + 4x^2 + px + q = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases} \quad \dots\dots\dots(1)$

$$x^3 + 6x^2 + px + r = 0 \begin{cases} \alpha \\ \beta \\ \delta \end{cases} \quad \dots\dots\dots(2)$$

(Subtract) _____

$$2x^2 + (r - q) = 0 \begin{cases} \alpha \\ \beta \end{cases}$$

So, $\alpha + \beta = 0 \Rightarrow \gamma = -4$ and $\delta = -6$.

$$\text{As, } x^2 + 2ax + 8c = 0 \begin{matrix} \nearrow \gamma \\ \searrow \delta \end{matrix}$$

$$\text{So, } \gamma + \delta = -2a \Rightarrow a = 5; \gamma\delta = 8c \Rightarrow c = 3$$

$$\text{Also, } \frac{\alpha\beta\gamma}{\alpha\beta\delta} = \frac{-q}{-r} \Rightarrow \frac{2}{3} = \frac{q}{r} \Rightarrow 3q = 2r$$

Note: If p, q, r all are zero, then option (D) is also correct.

Q.10 If $x^2 + 3x + 5$ is the greatest common divisor of $(x^3 + ax^2 + bx + 1)$ and $(2x^3 + 7x^2 + 13x + 5)$ then find the value of $[a + b]$.

[Note: $[k]$ denotes greatest integer less than or equal to k.]

Sol. $(x^3 + ax^2 + bx + 1) - (2x^3 + 7x^2 + 13x + 5) = (2a - 7)x^2 + (2b - 13)x - 3$

$$\text{Now, } (2a - 7)x^2 + (2b - 13)x - 3 = 0$$

$$\text{and } x^2 + 3x + 5 = 0$$

have both roots common

$$\frac{2a-7}{1} = \frac{2b-13}{3} = \frac{-3}{5}$$

$$2a - 7 = \frac{-3}{5} \Rightarrow a = \frac{35-3}{10} = 3.2$$

$$2b - 13 = \frac{-9}{5} \Rightarrow b = \frac{56}{10} = 5.6$$

DPP-4

SOLUTIONS

Q.1 If $x \in \mathbb{R}$ then range of $f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$ is

- (A) $(-\infty, \infty)$ (B) $\mathbb{R} - \left\{\frac{1}{2}\right\}$ (C) $\mathbb{R} - \left\{\frac{4}{9}, \frac{1}{2}\right\}$ (D) $\mathbb{R} - \left\{\frac{3}{2}\right\}$

Sol. $f(x) = \frac{(x+3)(x-1)}{(2x-3)(x+3)}$

$$x \neq -3, \frac{3}{2}$$

$$f(x) = y = \frac{x-1}{2x-3}$$

$$2xy - 3y = x - 1$$

$$x = \frac{3y-1}{2y-1}$$

$$\therefore y \neq \frac{1}{2}$$

$$\text{but since } x \neq -3 \text{ hence } y \neq \frac{4}{9}.$$

$$\therefore y \in \mathbb{R} - \left\{\frac{4}{9}, \frac{1}{2}\right\}$$

Q.2 If the highest point on the graph of $y = -x^2 - 2kx + 3a$ is $(-1, 2)$ then the value of $(k + 6a)$ is
(A) 2 (B) 3 (C) 5 (D) 6

Sol. Since $y = -x^2 - 2kx + 3a$ is a parabola opening downward parabola, hence vertex is the highest point on the graph

$$\frac{-(-2k)}{2} = -1 \Rightarrow k = 1$$

$$\frac{-D}{4a} = 2$$

$$\frac{4(-1)(3a) - (4K^2)}{-4} = 2$$

$$12a + 4 = 8$$

$$12a = 4$$

$$a = \frac{1}{3}$$

$$\text{Hence, } (K + 6a) = 3]$$

Q.3 If the quadratic polynomial $f(x) = (a - 3)x^2 - 2ax + 3a - 7$ ranges from $[-1, \infty)$ for every $x \in \mathbb{R}$, then the value of a lies in

- (A) $[0, 2]$ (B) $[3, 5]$ (C) $[4, 6]$ (D) $[5, 7]$

(MATHEMATICS)

SPECIAL DPP

Sol. $f(x) = (9 - 3)x^2 - 2ax + 3a - 7$
Since range of $f(x) \in [-1, \infty) \forall x \in \mathbb{R}$

Hence range of $(a - 3)x^2 - 2ax + 3a - 6 \in [0, \infty)$

$\therefore a - 3 > 0$ and $D = 0$

on solving we get $a = 6$ only]

Consider, $P(t) = \frac{t^2 + 4t + 10}{t^2 + 4t + 5}, t \in \mathbb{R}$

and $Q(x) = x^2 - 2mx + 6m - 41$, where $x, m \in \mathbb{R}$.

Q.4 The sum of all integral values in the range of $P(t)$ is

- (A) 19 (B) 20 (C) 21 (D) 22

Q.5 If $Q(x) + 54 \geq P(t) \forall x \in \mathbb{R}$ then true set of values of m is

- (A) $[-1, 6]$ (B) $[-6, 1]$ (C) $[-7, 1]$ (D) $[-1, 7]$

Sol. $P(t) = \frac{t^2 + 4t + 10}{t^2 + 4t + 5} = 1 + \frac{5}{(t+2)^2 + 1}$

(i) Sum of integral values of $P(x) = 2 + 3 + 4 + 5 + 6 = 20$. Ans.

(ii) $Q(x) + 54 \geq P(t) \forall x \in \mathbb{R}$

$$\Rightarrow x^2 - 2mx + 6m - 41 + 54 \geq 0$$

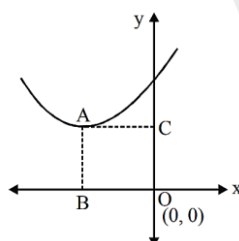
$$\Rightarrow x^2 - 2mx + 6m + 7 \geq 0 \forall x \in \mathbb{R} \text{ [As, maximum value of } P(t) \text{ is 6.]}$$

$$\text{Put } D \leq 0$$

$$\Rightarrow 4m^2 - 4(6m + 7) \leq 0$$

$$\Rightarrow m^2 - 6m - 7 \leq 0 \Rightarrow (m - 7)(m + 1) \leq 0 \Rightarrow -1 \leq m \leq 7. \text{ Ans.]}$$

The graph of $f(x) = ax^2 + bx + c$ is given, for which $l(AB) = 4, l(AC) = 4$ and $b^2 - 4ac = -8$.



Q.6 The value of $f(0) + 2f(1)$ is equal to

- (A) 45 (B) 26 (C) 24 (D) 20

Q.7 If α be one of the root of $f(x) = 0$ then the value of $(\alpha^3 + 10\alpha^2 + 40\alpha + 39)$ is

- (A) 0 (B) 9 (C) 10 (D) -9

(MATHEMATICS)

SPECIAL DPP

- Q.8** Range of $h(x) = \left(\frac{3}{2} + a\right)x^2 + (b-1)x + (c-6)$ when $x \in [-2, 0]$ is
 (A) $\left[\frac{39}{8}, 6\right]$ (B) $\left[\frac{39}{8}, 8\right]$ (C) $[6, 8]$ (D) $\left[\frac{39}{8}, \infty\right)$

Sol. $\frac{-D}{4a} = 4 \Rightarrow \frac{8}{4a} = 4 \Rightarrow a = \frac{1}{2}$

$\frac{-b}{2a} = -4 \Rightarrow b = 8a \Rightarrow b = 4$

$b^2 - 4ac = -8 \Rightarrow 16 - 2c = -8 \Rightarrow c = 12$

$f(x) = \frac{x^2}{2} + 4x + 12$ (i) $f(0) = 12$; $f(1) = \frac{33}{2}$

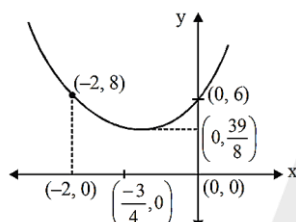
$f(0) + 2f(1) = 12 + 33 = 45$ Ans.

(ii) $f(x) = 0$ has root α

$\therefore \alpha^2 + 8\alpha + 24 = 0$

Hence, value of $(\alpha^3 + 10\alpha^2 + 40\alpha + 39)$ is -9 Ans.

(iii) $h(x) = 2x^2 + 3x + 6$



Range of $h(x)$ is $\left[\frac{39}{8}, 8\right]$

- Q.9** If $\sin^2 x + \sin x = (a+2)$, then which of the following statement(s) is(are) correct?

(A) Number of integral values of a for real solution to exist is 3.

(B) There exists no solution for $a < \frac{-9}{4}$ or $a > 0$.

(C) The minimum value of a for real solution is -2.

(D) Number of prime values of a for real solution to exist is 1.

Sol. $\sin^2 x + \sin x = (a+2)$

$\therefore y = \sin^2 x + \sin x = \left(\sin x + \frac{1}{2}\right)^2 - \frac{1}{4} \Rightarrow y \in \left[\frac{-1}{4}, 2\right]$

So, $\frac{-1}{4} \leq a+2 \leq 2 \Rightarrow \frac{-9}{4} \leq a \leq 0$ if $a \in \mathbb{I} \Rightarrow a = -2, -1, 0$.

- Q.10** Let $f(x) = x^2 + bx + c$ ($b, c \in \mathbb{R}$) for all $x \in \mathbb{R}$, attains its least value at $x = -1$ and the graph of $f(x)$ cuts y -axis at $y = 2$. Then

(A) the least value of $f(x)$ for all $x \in \mathbb{R}$ is 1.

(B) the value of $f(-2) + f(0) + f(1)$ equals 9.

(C) the least value of $f(x)$ for all $x \in \mathbb{R}$ is -1.

(D) the value of $f(-2) + f(0) + f(1)$ equals 7.

Sol. $f'(x) = 0$ at $x = -1 \Rightarrow b = 2$

and curve satisfy $(0, 2) \Rightarrow c = 2$

and $f(x) = (x+1)^2 + 1$

DPP-5

SOLUTIONS

Q.1 Let $P(x) = x^3 - 6x^2 + Bx + C$ has $1 + 5i$ as a zero and B, C are real numbers, then value of $(B + C)$ is

- (A) -70 (B) 70 (C) 24 (D) 138

Sol. $P(x) = x^3 - 6x^2 + Bx + C$ has roots $1 + 5i$ and $1 - 5i$, let third root $= \alpha$

$$\text{Sum of roots} = \frac{-b}{a} \Rightarrow 1 + 5i + 1 - 5i + \alpha = \frac{6}{1} \Rightarrow \alpha = 4$$

$$\text{Product of roots} = -C \Rightarrow (1 + 5i)(1 - 5i)\alpha = -C \Rightarrow 26 \cdot 4 = -C \Rightarrow C = -104$$

$$\text{So, } P(x) = x^3 - 6x^2 + Bx - 104$$

$$P(4) = 0 \Rightarrow 64 - 96 + 4B - 104 = 0 \Rightarrow B = 34$$

$$\text{Hence, } (B + C) = 34 - 104 = -70. \text{ Ans.}]$$

Q.2 If the equation $x^3 + 2x^2 - 4x + 5 = 0$ has roots α, β and γ , then the value of $\frac{(\alpha^3+5)(\beta^3+5)(\gamma^3+5)}{13\alpha\beta\gamma}$ is equal to

- (A) 5 (B) 8 (C) 12 (D) 15

Sol. $x^3 + 2x^2 - 4x + 5 \equiv (x - \alpha)(x - \beta)(x - \gamma)$

Put $x = 2$, we get

$$13 = (2 - \alpha)(2 - \beta)(2 - \gamma) \quad \dots\dots(1)$$

$$\text{Also, } \frac{(\alpha^3 + 5)(\beta^3 + 5)(\gamma^3 + 5)}{13\alpha\beta\gamma} = \frac{2\alpha \cdot (2 - \alpha) \cdot 2\beta \cdot (2 - \beta) \cdot 2\gamma \cdot (2 - \gamma)}{13\alpha\beta\gamma} = \frac{8}{13}(13) = 8 \text{ Ans.]}$$

Q.3 Let α, β, γ and δ be the roots (real or non-real) of equation $x^4 - 3x + 1 = 0$. The value of $\alpha^3 + \beta^3 + \gamma^3 + \delta^3$ is equal to

- (A) 6 (B) 9 (C) 12 (D) 15

Sol. $x^4 = 3x - 1, \sum \alpha = 0, \sum \alpha\beta = 0, \sum \alpha\beta\gamma = 3; \alpha\beta\gamma\delta = 1$

$$\therefore x^3 = 3 - \frac{1}{x} \begin{matrix} \nearrow \alpha \\ \nearrow \beta \\ \nearrow \gamma \\ \nearrow \delta \end{matrix}$$

$$\alpha^3 = 3 - \frac{1}{\alpha}; \beta^3 = 3 - \frac{1}{\beta}; \gamma^3 = 3 - \frac{1}{\gamma}; \delta^3 = 3 - \frac{1}{\delta}$$

on addin

$$\sum \alpha^3 = 12 - \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}\right) = 12 - \frac{\sum \alpha\beta\gamma}{\alpha\beta\gamma\delta} = 12 - 3 = 9 \text{ Ans.]}$$

Q.4 If the general expression of degree 2 given by $3x^2 + xy + ky^2 + 10x - 3y + 7$ can be factorised into two linear factors then value of k is

- (A) 4 (B) 2 (C) -2 (D) -4

(MATHEMATICS)

SPECIAL DPP

Sol. $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0]$

Q.5 If α, β, γ are roots of cubic equation $x^3 - 3x^2 + 2x + 4 = 0$ and

$y = 1 + \frac{\alpha}{x-\alpha} + \frac{\beta x}{(x-\alpha)(x-\beta)} + \frac{\gamma x^2}{(x-\alpha)(x-\beta)(x-\gamma)}$ then the value of y at $x = 2$, is

- (A) 0 (B) 1 (C) 2 (D) 3

Sol. $y = \frac{x^3}{(x-\alpha)(x-\beta)(x-\gamma)}$

$\therefore y(x=2) = \frac{8}{(2-\alpha)(2-\beta)(2-\gamma)} = \frac{8}{4} = 2$ Ans.

As, $(x^3 - 3x^2 + 2x + 4) = (x - \alpha)(x - \beta)(x - \gamma)$

$\therefore$ Put $x = 2$, we get $4 = (2 - \alpha)(2 - \beta)(2 - \gamma)$

For some real value of k , let $3\alpha^3 - \alpha^2 = k\alpha - 9, 3\beta^3 - \beta^2 = k\beta - 9, 3\gamma^3 - \gamma^2 = k\gamma - 9$ where $\alpha > \beta > \gamma$ and $\alpha + \gamma = 0$.

Q.6 The value of k is equal to

- (A) 27 (B) 0 (C) -9 (D) 3

Q.7 The minimum value of $f(x) = x^2 + \alpha x + \beta, x \in \mathbb{R}$ is equal to

- (A) $\frac{-23}{3}$ (B) $\frac{23}{12}$ (C) $\frac{-23}{12}$ (D) $\frac{23}{3}$

Q.8 The value of expression $(\alpha^{-2} + \beta^{-2} + \gamma^{-2})$ is equal to

- (A) $\frac{83}{9}$ (B) $\frac{84}{9}$ (C) $\frac{82}{9}$ (D) $\frac{85}{9}$

Sol. Consider an equation in x as

$3x^3 - x^2 - kx + 9 = 0$ (i)

$\begin{array}{c} \swarrow \quad \downarrow \quad \searrow \\ \alpha \quad \beta \quad \gamma \end{array}$

Now, $\alpha + \beta + \gamma = \frac{1}{3}$

$\Rightarrow \beta = \frac{1}{3}$ [As, $\alpha + \gamma = 0$ (given)]

$\therefore \beta = \frac{1}{3}$ will satisfy the equation (1)

(i) $\Rightarrow 3\left(\frac{1}{27}\right) - \frac{1}{9} - \frac{k}{3} + 9 = 0$

$\Rightarrow k = 27$

$\therefore$ Equation (1) becomes

$3x^3 - x^2 - 27x + 9 = 0 \Rightarrow (3x^2 - 27x) - (x^2 - 9) = 0$

(MATHEMATICS)

SPECIAL DPP

$$\Rightarrow 3x(x^2 - 9) - (x^2 - 9) = 0 \Rightarrow (3x - 1)(x + 3)(x - 3) = 0$$

$$\therefore x = -3, \frac{1}{3}, 3$$

$$\text{So, } \alpha = 3; \beta = \frac{1}{3}; \gamma = -3$$

$$(ii) \text{ Now, } f(x) = x^2 + 3x + \frac{1}{3} = \left(x + \frac{3}{2}\right)^2 + \frac{1}{3} - \frac{9}{4} = \left(x + \frac{3}{2}\right)^2 + \frac{4-27}{12} = \left(x + \frac{3}{2}\right)^2 - \frac{23}{12}$$

$$\therefore f_{\min.} \left(x = -\frac{3}{2}\right) = -\frac{23}{12} \text{ Ans.}$$

$$(iii) \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{1}{9} + 9 + \frac{1}{9} = \frac{2}{9} + 9 = \frac{83}{9} \text{ Ans.]}$$

Q.9 Let roots of the equation $x^3 + 3x^2 + 4x = 11$ are α, β, γ and the roots of equation $x^3 + lx^2 + mx + n = 0$ ($l, m, n \in \mathbb{R}$) are $\alpha + \beta, \beta + \gamma, \gamma + \alpha$.

Column-I

- (A) l is equal to
- (B) m is equal to
- (C) n is equal to
- (D) $(l + m + n)$ is equal to

Column-II

- (P) -6
- (Q) 6
- (R) 13
- (S) 23
- (T) 42

Sol. Given, $x^3 + 3x^2 + 4x - 11 = 0$ $\begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

$$x^3 + lx^2 + mx + n = 0 \begin{cases} \alpha + \beta \\ \beta + \gamma \\ \gamma + \alpha \end{cases}$$

$$\text{Here, } \alpha + \beta + \gamma = -3; \alpha\beta + \beta\gamma + \gamma\alpha = 4; \alpha\beta\gamma = 11$$

$$\text{Also, } (\alpha + \beta) + (\beta + \gamma) + (\gamma + \alpha) = -l$$

$$\Sigma(\alpha + \beta)(\beta + \gamma) = m$$

$$\text{and } (\alpha + \beta)(\beta + \gamma)(\gamma + \alpha) = -n$$

$$\therefore \text{ We get, } l = 6, m = 13 \text{ and } n = 23$$

$$\Rightarrow (l + m + n) = 42. \text{ Ans.]}$$

Q.10 If the system of equation $r^2 + s^2 = t$ and $r + s + t = \frac{k-3}{2}$ has exactly one real solution, then find the value of k .

Sol. $r + s + r^2 + s^2 = \frac{k-3}{2}$

$$r^2 + r + s^2 + s + \frac{3-k}{2} = 0$$

$$D = 0$$

$$1 - 4\left(s^2 + s + \frac{3-k}{2}\right) = 0 \Rightarrow 4s^2 + 4s + 5 - 2k = 0$$

$$D = 0$$

$$16 - 16(5 - 2k) = 0 \Rightarrow 1 = 5 - 2k \Rightarrow 2k = 4$$

$$\therefore k = 2$$

DPP-6

SOLUTIONS

Q.1 The range of $p \in \mathbb{R}$ for which the equation $2x^2 - 2(2p + 1)x + p(p + 1) = 0$ have one root less than p and other root greater than p , is

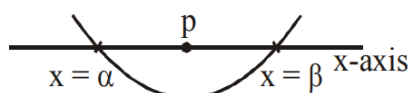
(A) $-1 < p < 0$

(B) $p < -1$ or $p > 0$

(C) $p \geq 0$

(D) $p = 0$

Sol. We must have $f(p) < 0$, where $f(x) = 2x^2 - 2(2p + 1)x + p(p + 1)$



So, $f(p) < 0 \Rightarrow p^2 + p > 0 \Rightarrow p > 0$ or $p < -1$.]

Q.2 If exactly one root of the quadratic equation $x^2 - \left(k + \frac{11}{3}\right)x - (k^2 + k + 1) = 0$ lies in $(0, 3)$ then which one of the following relation is correct?

(A) $-8 < k < -4$

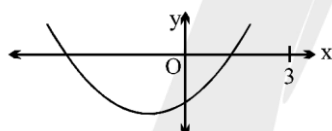
(B) $-3 < k < -1$

(C) $1 < k < 4$

(D) $6 < k < 10$

Sol. We have $x^2 + \left(k + \frac{11}{3}\right)x - (k^2 + k + 1) = 0$

Product of roots $= -(k^2 + k + 1)$, which is negative. $\forall k \in \mathbb{R}$ Hence the only graph which is possible is as shown. (No need to check at end points.)



Now $f(0) < 0$ and $f(3) > 0 \Rightarrow 9 - \left(k + \frac{11}{3}\right)3 - k^2 - k - 1 > 0 \Rightarrow -k^2 - 4k - 3 > 0$

$\Rightarrow k^2 + 4k + 3 < 0 \Rightarrow k \in (-3, -1)$]

Q.3 If the equation $x^2 + ax + b = 0$ has one root equal to unity and other root lies between roots of the equation $x^2 - 7x + 12 = 0$, then the range of a is

(A) $(-5, -4)$

(B) $(-4, -3)$

(C) $(-3, -2)$

(D) $(4, 5)$

Sol. Two roots are 1, b . So, $1 + a + b = 0 \Rightarrow b = -(1 + a)$ [As $\alpha\beta = b$ and $\alpha = 1$; so $\beta = b$]

Also, $3 < b < 4$ (given) $\Rightarrow 3 < -(1 + a) < 4 \Rightarrow -5 < a < -4$. Ans.]

Q.4 The greatest integral value of p for which the quadratic equation $x^2 - p(2x - 8) = 15$ has one root less than 1 and other root greater than 2, is

(A) -1

(B) 0

(C) 1

(D) 2

(MATHEMATICS)

SPECIAL DPP

Sol. Let $f(x) = x^2 - 2px + 8p - 15$

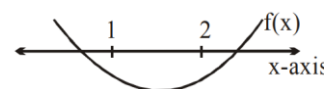
Clearly 1 and 2 lies between the roots.

Now, $f(1) < 0 \Rightarrow 6p - 14 < 0 \Rightarrow p < \frac{7}{3}$ (1)

and $f(2) < 0 \Rightarrow 4p - 11 < 0 \Rightarrow p < \frac{11}{4}$ (2)

$\therefore$ From $(1) \cap (2)$, we get $p < \frac{7}{3}$

Hence, the greatest integral value of $p = 2$. Ans.]



Q.5 Let $P(x) = x^2 - 2(a^2 + a + 1)x + a^2 + 5a + 2$. If minimum value of $P(x)$ for $x \leq 0$ is 8, then the sum of the squares of all possible value(s) of a , is

- (A) 13 (B) 17 (C) 37 (D) 49

Sol. SC Vertex: $x = (a^2 + a + 1) > 0$

Hence, $P(x)$ is decreasing in $(-\infty, 0)$

$\therefore$ Minimum value occur at $x = 0$

hence $f(0) = 8$

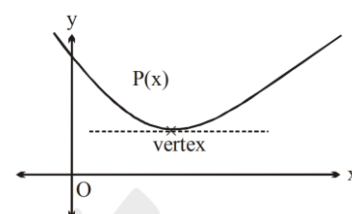
$\Rightarrow a^2 + 5a + 2 = 8$

$\Rightarrow a^2 + 5a - 6 = 0$

$\Rightarrow (a + 6)(a - 1) = 0$

$\therefore a = 1$ or -6

Hence $(1)^2 + (-6)^2 = 1 + 36 = 37$. Ans.]



Q.6 If $\lambda, p \in \mathbb{R}$ and $p \in [-5, 10]$ then the number of integral value of 'p' for which $e^\lambda + 1$ and $e^{-\lambda} + 1$ are the roots of the quadratic equation $x^2 + (1 - 2p)x + 2p - 1 = 0$ is

- (A) 7 (B) 8 (C) 14 (D) 16

Sol. SC $D \geq 0 \Rightarrow (2p - 1)^2 - 4(2p - 1) \geq 0 \Rightarrow (2p - 1)(2p - 5) \geq 0 \Rightarrow p \in (-\infty, \frac{1}{2}] \cup [\frac{5}{2}, \infty)$

$\alpha + \beta = e^\lambda + 1 + e^{-\lambda} + 1 \geq 4 \Rightarrow 2p - 1 \geq 4 \Rightarrow p \geq \frac{5}{2}$

$\alpha\beta = (e^\lambda + 1)(e^{-\lambda} + 1) = 1 + e^\lambda + e^{-\lambda} + 1 \geq 4 \Rightarrow 2p - 1 \geq 4 \Rightarrow p \geq \frac{5}{2}$

$\alpha + \beta = \alpha\beta \Rightarrow 2p - 1 = 2p - 1 \Rightarrow p \in \mathbb{R}$

$\therefore p \in [\frac{5}{2}, \infty)$

Number of values of 'p' in $[-5, 10]$ are 8.]

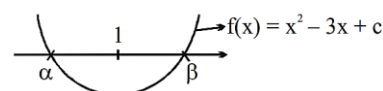
Q.7 If α, β are the roots of equation $x^2 - 3x + c = 0 (c \in \mathbb{R})$ and $\alpha < 1 < \beta$, then c belongs to

- (A) $(-\infty, \frac{9}{4})$ (B) $(-\infty, 2)$ (C) $(2, \infty)$ (D) $(\frac{9}{4}, \infty)$

Sol. $\therefore f(1) < 0$

$\Rightarrow 1 - 3 + c < 0$

$\Rightarrow c < 2$

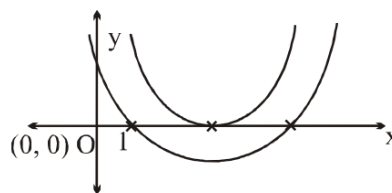


(MATHEMATICS)

SPECIAL DPP

Q.8 Let $f(x) = x^2 - 6kx + k^2 + 6k$, $k \in [-5, 5]$ and $x \in \mathbb{R}$. If both roots of the equation $f(x) = 0$ are greater than unity and minimum integral value of $f(1)$ is β , then

- (A) $\beta = 1$
 (B) $\beta = 2$
 (C) number of integral values of k is 4.
 (D) number of integral values of k is 5.



Sol. $f(x) = x^2 - 6kx + k^2 + 6k$, $k \in [-5, 5]$

$$\text{Here, } \frac{6k}{2} > 1 \Rightarrow k > \frac{1}{3} \dots\dots(1)$$

$$f(1) > 0 \Rightarrow 1 + k^2 > 0 \dots\dots(2)$$

$$\text{Also, } D \geq 0 \Rightarrow k \leq 0 \text{ or } k \geq \frac{3}{4}$$

$$\therefore (1) \cap (2) \cap (3) \dots\dots(3)$$

$$\Rightarrow k \geq \frac{3}{4}$$

$$\therefore k \text{ integers} = 1, 2, 3, 4, 5$$

$$\text{Also } f(1) = (1 + k^2)_{\min.} = 1 + (1)^2 = 2 \text{ (for } k = 1 \text{)]}$$

Q.9 If $\alpha + \frac{1}{\alpha}$ and $2 - \beta - \frac{1}{\beta}$ ($\alpha, \beta > 0$) are the roots of the quadratic equation

$$x^2 - 2(a + 1)x + a - 3 = 0 \text{ then find the sum of integral values of 'a' .}$$

Sol. $\alpha + \frac{1}{\alpha} \geq 2$

$$\beta + \frac{1}{\beta} \Rightarrow -\beta - \frac{1}{\beta} \leq -2 \Rightarrow 2 - \beta - \frac{1}{\beta} \leq 0$$

$\therefore$ one root is greater than or equal to 2 and other root is smaller than or equal to 0.

$$x^2 - 2(a + 1)x + a - 3 = 0$$

$$(i) D \geq 0$$

$$4(a + 1)^2 - 4(a - 3) \geq 0 \Rightarrow a^2 + 2a + 1 - a + 3 \geq 0 \Rightarrow a^2 + a + 4 \geq 0 \Rightarrow a \in \mathbb{R}$$

$$(ii) f(0) \leq 0 \Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$$

$$(iii) f(2) \leq 0 \Rightarrow 4 - 4(a + 1) + a - 3 \leq 0 \Rightarrow -3a - 3 \leq 0 \Rightarrow a \geq -1$$

$$\therefore a \in [-1, 3]$$

$$\text{Required sum} = -1 + 0 + 1 + 2 + 3 = 5 \text{ Ans.]}$$

Q.10 If α, β are the roots of the quadratic equation,

$$(a^2 - 4a + 3)x^2 - (a^3 - 8a - 1)x + \log_{1/2}(a^2 - 6a + 9) = 0$$

such that $\alpha < 0 < \beta$ then the range of a is $(-\infty, p) \cup (q, r) \cup (s, \infty)$. Find the value of $(p + q + r - s)$.

Sol. If coefficient of x^2 i.e. $a^2 - 4a + 3 > 0$ i.e. $(a - 3)(a - 1) > 0$ i.e. $a > 3$ or $a < 1$

Hence solution set is

$$(-\infty, 1) \cup (4, \infty)$$

If $(a - 3)(a - 1) < 0$ i.e. $1 < a < 3$

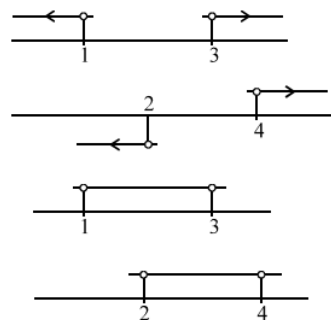
then $f(0) > 0$

$$\therefore (a - 4)(a - 2) < 0$$

Hence the solution set is $(2, 3)$

from (1) and (2)

$$a \in (-\infty, 1) \cup (2, 3) \cup (4, \infty) \text{Ans.]}$$



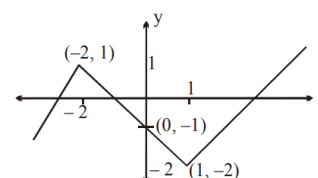
SOLUTIONS

Q.1 If the equation $|1 - x| - |x + 2| + x = p$ has two distinct real solutions then

- (A) $p \in (-2, 1)$ (B) $p \in [-2, 1]$ (C) $p \in \{-1, 2\}$ (D) $p \in \{-2, 1\}$

Sol. $y = |x - 1| - |x + 2| + x = \begin{cases} x - 3, & x \geq 1 \\ -x - 1, & -2 \leq x < 1 \\ x + 3, & x < -2 \end{cases}$

So, for 2 distinct real solution, $p \in \{-2, 1\}$



Q.2 The smallest integral value of α for which the inequality $1 + \log_5 (x^2 + 1) \leq \log_5 (\alpha x^2 + 4x + \alpha)$ is true for all $x \in \mathbb{R}$, is

- (A) 4 (B) 5 (C) 6 (D) 7

Sol. We have, $(5 - \alpha)x^2 - 4x + (5 - \alpha) \leq 0 \forall x \in \mathbb{R}$

$$\therefore 5 - \alpha < 0 \text{ and } \text{Disc.} \leq 0$$

$$\Rightarrow \alpha > 5 \text{ and } 16 \leq 4(\alpha - 5)^2$$

$$\Rightarrow \alpha > 5 \text{ and } (5 - \alpha)^2 - (2)^2 \geq 0$$

$$\Rightarrow \alpha > 5 \text{ and } (\alpha - 7)(\alpha - 3) \geq 0$$

So, $\alpha \in [7, \infty)$ Ans.]

Q.3 The number of integral solution(s) of the inequality

$$\log_{(x^2+2)} (x^4 + x^2 + 2) \geq \sqrt{2 - \log_3 (3 + |x|)} \text{ is}$$

- (A) 0 (B) 1 (C) 6 (D) 13

Sol. $\log_{(x^2+2)} (x^4 + x^2 + 2) \geq \sqrt{2 - \log_3 (3 + |x|)}$

$$2 - \log_3 (3 + |x|) \geq 0 \Rightarrow \log_3 (3 + |x|) \leq 2$$

$$3 + |x| \leq 9 \Rightarrow x \in [-6, 6]$$

Q.4 If the equation $|x^2 + 2x + a| = 2$ has exactly 4 real and distinct solutions, then

- (A) $a > 3$ (B) $a \in (-\infty, -1] \cup (2, \infty)$
(C) $a \in (-\infty, 1) \cup (3, \infty)$ (D) $a < -1$

Sol. $x^2 + 2x + a = \pm 2$

$$x^2 + 2x \pm 2 = -a$$

$$x^2 + 2x + 2 + a = 0 \quad \text{or} \quad x^2 + 2x + a - 2 = 0$$

$$D_1 > 0 \quad \& \quad D_2 > 0$$

$$4 - 8 - 4a > 0 \quad \& \quad 4 - 4a + 8 > 0 \quad a + 1 < 0 \quad \& \quad a < 3$$

$$a < -1 \quad \& \quad a < 3 \quad \therefore a < -1.$$

(MATHEMATICS)

SPECIAL DPP

Q.5 The number of integral value(s) of x satisfying both the inequalities $\log_{\sqrt{5}} (4 - x) \geq 0$ and $\log_{(x-1)} (x^2 + 2) \geq 0$ is

- (A) 1 (B) 2 (C) 3 (D) 4

Sol. $\log_{\sqrt{5}} (4 - x) \geq 0$

$$4 - x > 0 \text{ and } 4 - x \geq 1$$

$$\therefore x \in (-\infty, 3]$$

$$\log_{(x-1)} (x^2 + 2) \geq 0$$

Case-I : $x - 1 > 1 \Rightarrow x > 2$

$$x^2 + 2 \geq 1 \Rightarrow x^2 \geq -1$$

$$\therefore x \in (2, \infty)$$

Case-II : $0 < x - 1 < 1$

This case is not possible.

Therefore from case-I and case-II

$$\therefore x \in (2, \infty)$$

Hence, from (1) and (2)

$$x \in (2, 3]]$$

Q.6 For $2 < x < 5$, the expression $|2x - 1| + |5 - 3x| + |x - 6| = Ax + B$. The value of $A + B$, is

- (A) 2 (B) 4 (C) 8 (D) 10

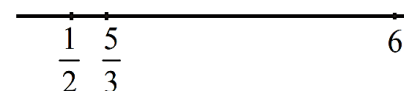
Sol. $2 < x < 5$

$$|2x - 1| + |5 - 3x| + |x - 6| = Ax + B$$

$$2x - 1 - 5 + 3x - x + 6 = Ax + B$$

$$4x + 0 = Ax + B$$

$$A + B = 4. \text{ Ans.]}$$



Q.7 Column-I Column-II
(A) The possible integer(s) satisfying the inequality (P) 2

$$\log_{(2x-3)} (3x - 4) > 0, \text{ is}$$

(B) The possible integer(s) satisfying the inequality (Q) 3

$$\log_{\tan \frac{\pi}{6}} \left(\frac{x^2 - x + 1}{x + 2} \right) > -2, \text{ is}$$

(C) The possible integer(s) satisfying the inequality (R) 4
(S) 5

$$\log_{0.5} (x^2 - 3x + 4) - \log_{0.5} (x - 1) + 1 < 0, \text{ is}$$

(MATHEMATICS)

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Sol. (A) **Case-I :** If base > 1 i.e. $(2x - 3) > 1 \Rightarrow x > 2$

$$\text{Now, } (3x - 4) > 1 \Rightarrow x > \frac{5}{3}$$

Hence, $x \in (2, \infty)$ (i)

Case-II: When $0 < \text{base} < 1$ i.e. $0 < 2x - 3 < 1 \Rightarrow \frac{3}{2} < x < 2$

$$\text{Now, } 0 < 3x - 4 < 1 \Rightarrow \frac{4}{3} < x < \frac{5}{3}$$

Hence, $x \in \left(\frac{3}{2}, \frac{5}{3}\right)$ (2)

$\therefore$ Case I $\cup$ case II

$$\Rightarrow x \in \left(\frac{3}{2}, \frac{5}{3}\right) \cup (2, \infty) \Rightarrow 3, 4, \dots \dots \text{Ans.]}$$

$$(B) \log_{\frac{1}{\sqrt{3}}} \left(\frac{x^2 - x + 1}{x + 2} \right) > -2 \Rightarrow 0 < \frac{x^2 - x + 1}{x + 2} < 3$$

since $x^2 - x + 1 > 0$, hence domain is $x + 2 > 0$

$$\text{now } \frac{x^2 - x + 1}{x + 2} < 3$$

$$x^2 - x + 1 < 3x + 6 \Rightarrow x^2 - 4x - 5 < 0 \Rightarrow (x - 5)(x + 1) < 0$$

Hence $x \in (-1, 5)$ also domain is $x > -2$

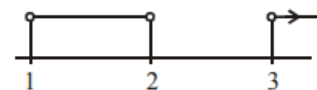
$$\Rightarrow -1 < x < 5 \Rightarrow 0, 1, 2, 3, 4 \text{ Ans.]}$$

$$(C) \log_{\frac{1}{2}} \frac{x^2 - 3x + 4}{(x - 1)} < -1$$

$$\Rightarrow \frac{x^2 - 3x + 4}{x - 1} > \left(\frac{1}{2}\right)^{-1} = 2$$

$$\frac{x^2 - 3x + 4 - 2x + 2}{(x - 1)} > 0 \text{ or } \frac{x^2 - 5x + 6}{x - 1} > 0 \text{ or } \frac{(x - 3)(x - 2)}{x - 1} > 0$$

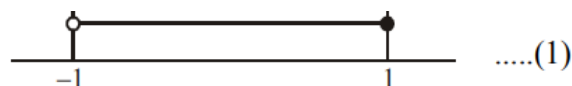
$$x \in (1, 2) \cup (3, \infty) \Rightarrow 4, 5, \dots \dots \text{Ans.]}$$



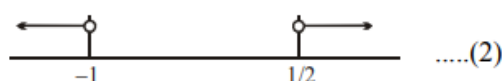
Q.8 Find the number of integral values of x satisfying the inequality $\log_4 \left(\frac{2x-1}{x+1} \right) \leq \cos \left(\frac{4\pi}{3} \right)$. [Ans. 1]

Sol. We have $\log_4 \left(\frac{2x-1}{x+1} \right) \leq \frac{-1}{2} \Rightarrow \log_2 \left(\frac{2x-1}{x+1} \right) \leq -1 \Rightarrow 0 < \frac{2x-1}{x+1} \leq \frac{1}{2}$

$$\text{Now, } \frac{2x-1}{x+1} - \frac{1}{2} \leq 0 \Rightarrow \frac{4x-2-x-1}{x+1} \leq 0 \Rightarrow \frac{x-1}{x+1} \leq 0 \Rightarrow x \in (-1, 1]$$



$$\text{and } \frac{2x-1}{x+1} > 0 \Rightarrow x \in (-\infty, -1) \cup \left(\frac{1}{2}, \infty\right)$$



$$\therefore (1) \cap (2) \Rightarrow \left(\frac{1}{2}, 1\right]$$

Hence number of integral values of x is 1.

(MATHEMATICS)

SPECIAL DPP

Q.9 Find the smallest value of $\left(\frac{-8p}{7}\right)$ for which $|x^2 - 5x + 7 - p| = 6 + |x^2 - 5x + 1 - p|$ for all $x \in [-1, 3]$.

Sol. We must have

$$6(x^2 - 5x + 1 - p) \geq 0 \quad \forall x \in [-1, 3]$$

$$\Rightarrow \underbrace{x^2 - 5x + (1 - p)}_{\text{upward parabola with vertex}} \geq 0 \quad \forall x \in [-1, 3]$$

$$\left(x = \frac{5}{2}, y = -p - \frac{21}{4}\right)$$

$$\therefore -p - \frac{21}{4} \geq 0 \Rightarrow p + \frac{21}{4} \leq 0 \Rightarrow p \leq \frac{-21}{4}$$

$$\therefore p \in \left(-\infty, \frac{-21}{4}\right]$$

$$\Rightarrow \text{the smallest value of } \left(\frac{-8p}{7}\right) = \frac{-8}{7} \times \frac{-21}{4} = 6. \text{ Ans.]}$$

Q.10 Find the number of integral solutions of the inequation

$$\log_9 (x + 1) \cdot \log_2 (x + 1) - \log_9 (x + 1) - \log_2 (x + 1) + 1 < 0.$$

Sol. We have $(\log_9 (x + 1) - 1)(\log_2 (x + 1) - 1) < 0$

$$\Rightarrow \left(\log_9 \left(\frac{x+1}{9}\right)\right) \left(\log_2 \left(\frac{x+1}{2}\right)\right) < 0$$

$$\text{Case-I: } \log_9 \left(\frac{x+1}{9}\right) > 0 \Rightarrow \frac{x+1}{9} > 1 \Rightarrow x+1 > 9 \Rightarrow x > 8$$

$$\text{and } \log_2 \left(\frac{x+1}{2}\right) < 0 \Rightarrow 0 < \frac{x+1}{2} < 1 \Rightarrow 0 < (x+1) < 2$$

$$\Rightarrow -1 < x < 1 \dots (2)$$

$$\therefore (1) \cap (2) \Rightarrow x \in \phi$$

$$\text{Case-II: } \log_9 \left(\frac{x+1}{9}\right) < 0 \Rightarrow 0 < \frac{x+1}{9} < 1 \Rightarrow 0 < x+1 < 9 \Rightarrow -1 < x < 8$$

$$\text{and } \log_2 \left(\frac{x+1}{2}\right) > 0 \Rightarrow \frac{x+1}{2} > 1 \Rightarrow x+1 > 2 \Rightarrow x > 1$$

$$\therefore (3) \cap (4) \Rightarrow x \in (1, 8)$$

$$\text{So, } x = 2, 3, 4, 5, 6, 7]$$